CHAPTER 1: THE CONCEPT OF STRESS

Introduction to Mechanics (Strength) of Materials: Objectives

The main objective of the study of the mechanics of materials is to investigate the behaviour of solid bodies subjected to various types of loading. We aim to determine whether or not a solid body can withstand the applied loads to it. An understanding of mechanical behaviour is essential for the safe design of all types of structures. The solid bodies considered in this course include bars with axial loads, shafts in torsion, and beams in bending. In this course only solid structures with simplistic geometries such as bodies with circular or square cross sections are considered.

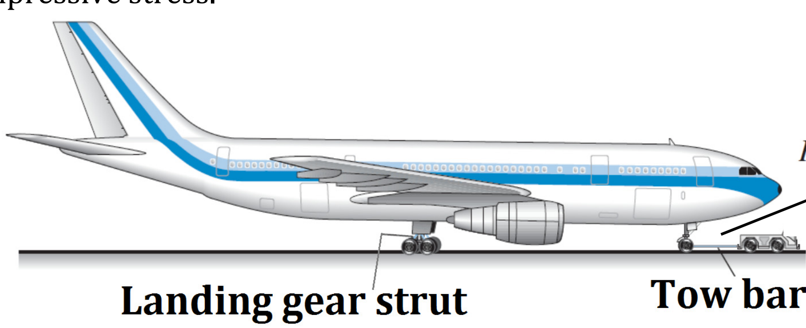
The difference of the study of "Statics" with "Mechanics of materials" is that in "Statics" we simply aim to determine internal loads that a structure undergoes but we do not investigate whether or not the structure can withstand the applied loads to it. In this course, however, we not only determine if the structure can withstand the applied loads or not but we estimate the deformation of the structure under the applied loads too. Determining deformations is a very important step toward the safe design of all types of structures (*Turbine failure of flight Qantas 32, A380*).

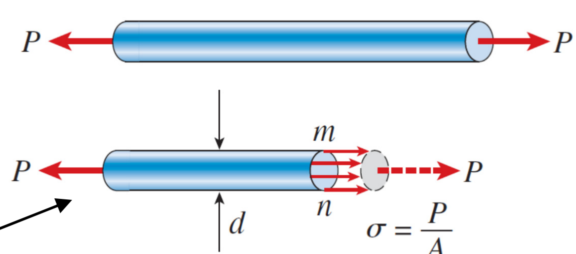
Concept of Stress (Normal Stress)

"Force" is not an appropriate quantity to describe the tolerance of materials. For example a steel bar with 1 mm² cross sectional area can withstand very small tensile or compressive forces compared to the same steel bar having a cross sectional area of 100 mm². Therefore, a quantity is defined which does not depend on the size of sample bar; force divided by cross sectional area:

$$\boldsymbol{\sigma} = \frac{\boldsymbol{F}}{\boldsymbol{A}} \qquad units: MPa = \frac{N}{mm^2} \quad or \quad Pa = \frac{N}{m} \qquad Sign: +\ for\ tensile\ and\ -\ for\ compressive$$

For example Aluminum Alloy (1100-H14) can tolerate 110 MPa tensile stress and 70 MPa compressive stress.

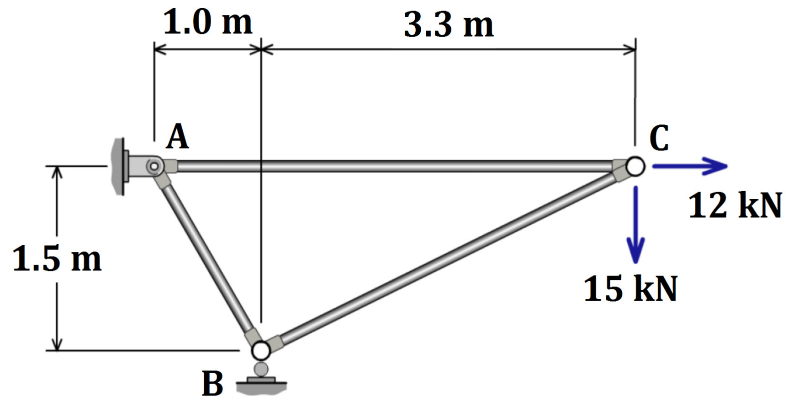




$$\sigma = \frac{P}{A} = \frac{P}{\frac{\pi}{4}d^2} = \frac{P}{\pi r^2}$$

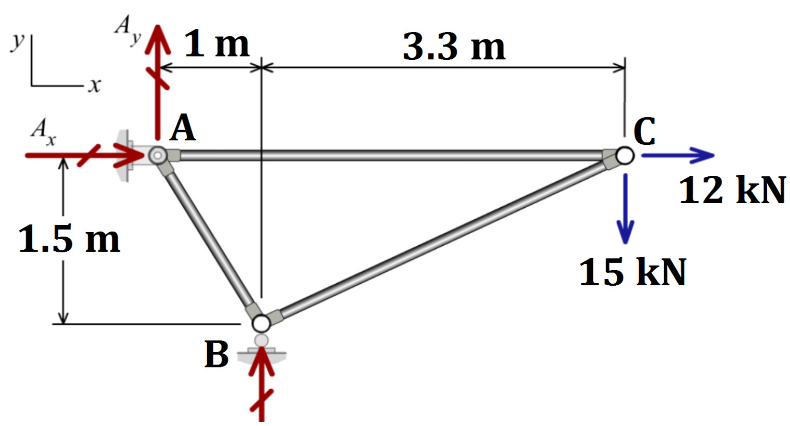
Structural members subjected to axial loads. The tow bar is in tension and the landing gear strut is in compression.

Example 1: A simple pin-connected truss is loaded and supported as shown. All members of the truss are aluminum pipes that have an outside diameter of 60 mm and a wall thickness of 4 mm. determine the normal stress in each truss member.



Overal equilibrium:

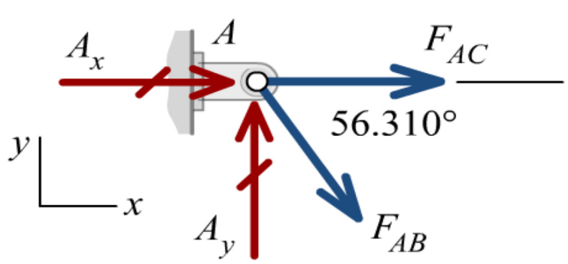
$$\Sigma F_x = A_x + 12\text{ kN} = 0$$
$$\therefore A_x = -12\text{ kN}$$
$$\Sigma M_A = B_y(1\text{ m}) - (15\text{ kN})(4.3\text{ m}) = 0$$
$$\therefore B_y = 64.5\text{ kN}$$
$$\Sigma F_y = A_y + B_y - 15\text{ kN} = 0$$
$$\therefore A_y = -49.5\text{ kN}$$



Equilibrium at joint A:

$$\Sigma F_x = F_{AC} + F_{AB}\cos(56.310°) + A_x = 0$$
$$\Sigma F_y = A_y - F_{AB}\sin(56.310°) = 0$$

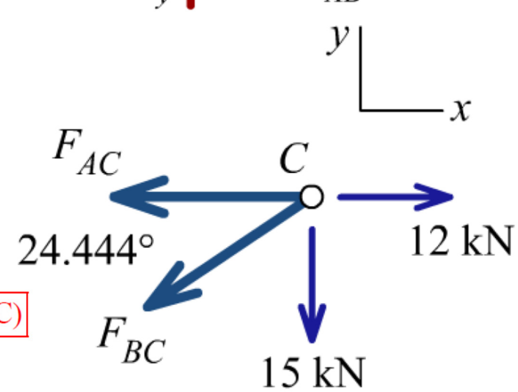
$$F_{AB} = -59.492\text{ kN}$$
$$F_{AC} = 45\text{ kN}$$



Equilibrium at joint C:

$$\Sigma F_x = -F_{AC} - F_{BC}\cos(24.444°) + 12\text{ kN} = 0$$
$$\Sigma F_y = -F_{BC}\sin(24.444°) - 15\text{ kN} = 0$$

$$F_{BC} = -36.249\text{ kN}$$



$$A = \frac{\pi}{4}\left[(60\text{ mm})^2 - (52\text{ mm})^2\right] = 703.7168\text{ mm}^2$$

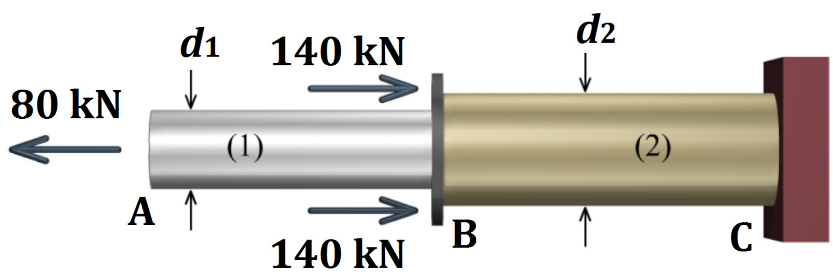
$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{(-59.492\text{ kN})(1{,}000\text{ N/kN})}{703.7168\text{ mm}^2} = -84.539\text{ MPa} = \boxed{84.5\text{ MPa (C)}}$$

$$\sigma_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{(45.000\text{ kN})(1{,}000\text{ N/kN})}{703.7168\text{ mm}^2} = 63.946\text{ MPa} = \boxed{63.9\text{ MPa (T)}}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{(-36.249\text{ kN})(1{,}000\text{ N/kN})}{703.7168\text{ mm}^2} = -51.511\text{ MPa} = \boxed{51.5\text{ MPa (C)}}$$

Does the structure fail? Where does it fail from? What to do to avoid its failure?

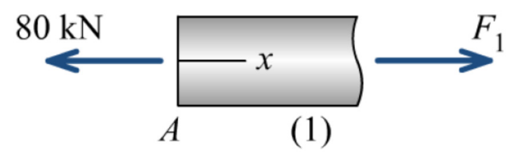
Example 2: Two solid cylindrical rods (1) and (2) are joined together at flange *B* and loaded, as shown. If the normal stress in each rod must be limited to 120 MPa, determine the minimum diameter required for each rod.



$$\Sigma F_x = F_1 - 80\text{ kN} = 0 \quad \therefore F_1 = 80\text{ kN (T)}$$

$$\Sigma F_x = F_2 + 140\text{ kN} + 140\text{ kN} - 80\text{ kN} = 0$$

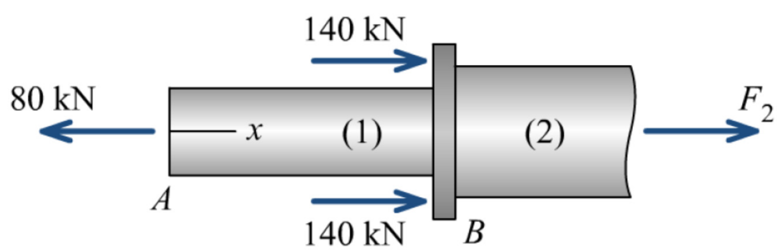
$$\therefore F_2 = -200\text{ kN} = 200\text{ kN (C)}$$



$$\sigma_1 = \frac{F_1}{A_1} \rightarrow 120\ MPa = \frac{80\,000\ N}{A_1} \rightarrow A_1 = 666.666\ mm^2$$

$$\rightarrow d_1 = 29.1\ mm$$

$$\sigma_2 = \frac{F_2}{A_2} \rightarrow 120\ MPa = \frac{200\,000\ N}{A_2}$$

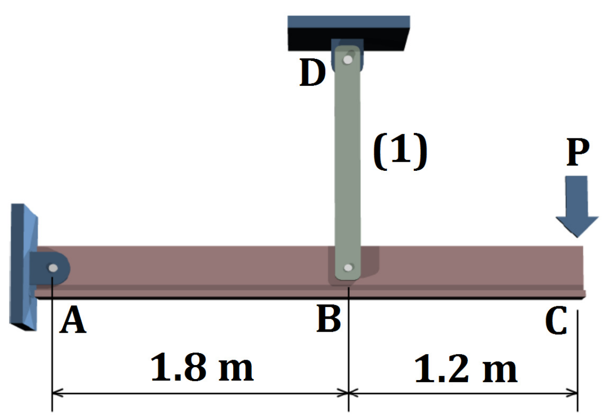


$$\rightarrow A_2 = 1666.666\ mm^2$$

$$\rightarrow d_2 = 46.1\ mm$$

It is evident that these are minimal values for d_1 and d_2.

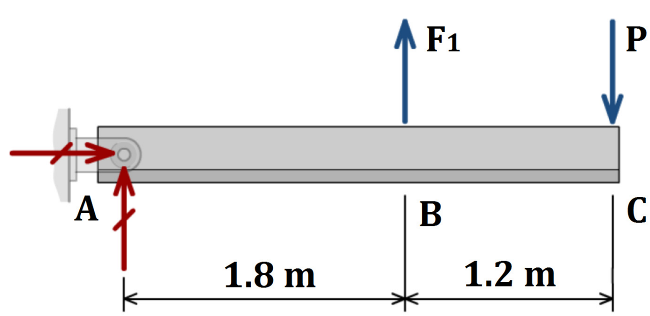
Example 3: Bar (1) has a cross sectional area of 485 mm². If the stress in bar (1) must be limited to 210 MPa, determine the maximum load *P* that may be supported by the structure (the rod ABC is rigid).



$$\sigma_1 = \frac{F_1}{A_1} \rightarrow 210\ MPa = \frac{F_1}{485\ mm^2} \rightarrow F_1 = 101850\ N$$

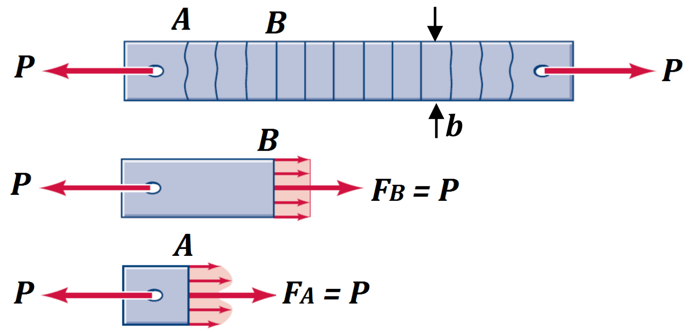
$$\sum M_A = 0 \rightarrow 3.0\ P - 1.8\ F_1 = 0 \rightarrow F_1 = 1.6667\ P$$

$$P = \frac{F_1}{1.6667} = \frac{101850\ N}{1.6667} = 61110\ N$$



Limitations of $\sigma = F/A$

The distribution of force on cross sections at A and B are shown. Near the ends of the bar, for example at section A, the resultant normal force, F_A, is not uniformly distributed over the cross section; but at section B, farther from the point of application of force P, the force distribution is uniform (***Saint-Venant's Principle***).



The distribution of internal force at section *A* and *B*

$Average\ stress: \sigma_{avg} = \dfrac{P}{A}$

$Stress\ at\ a\ point: \sigma = \lim\limits_{\Delta A \to 0} \dfrac{\Delta F}{\Delta A}$

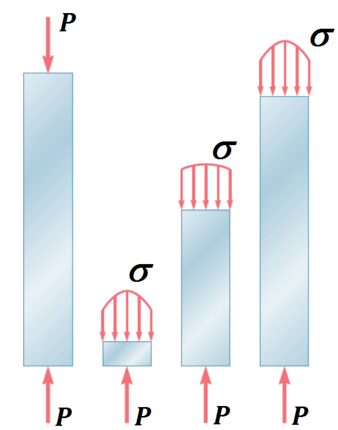
As a practical rule, the formula $\sigma = P/A$ may be used with *good accuracy* at any point within a prismatic bar that is at least as far away from the stress concentration as the largest lateral dimension of the bar (the stress distribution in the bar shown above is uniform at distances b or greater from the enlarged ends).

At section B: $\sigma_{avg} \cong \sigma_B\ (at\ all\ points)$ but at section A: $\sigma_{avg} \neq \sigma_A$

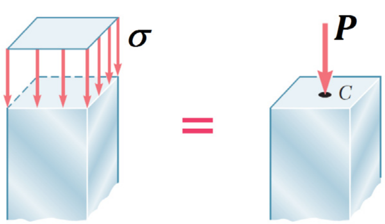
It is thus ***assumed*** that: $\boldsymbol{\sigma_{avg)B} = \frac{F_B}{A} = \frac{P}{A}}$ ***and*** $\boldsymbol{\sigma_{avg)A} = \frac{F_A}{A} = \frac{P}{A}}$

In other words:

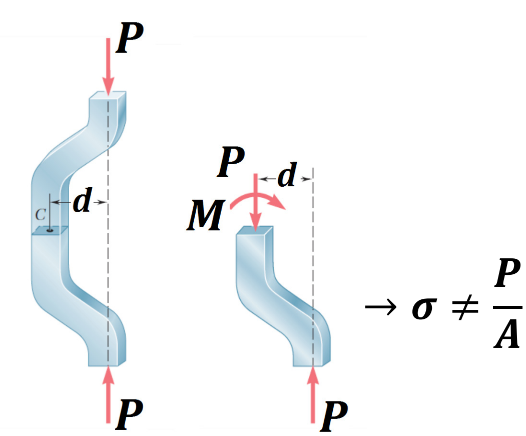
$$P = \int dF = \int_A \sigma\, dA \quad it\ is\ assumed\ that\ P = \sigma_{avg} A$$



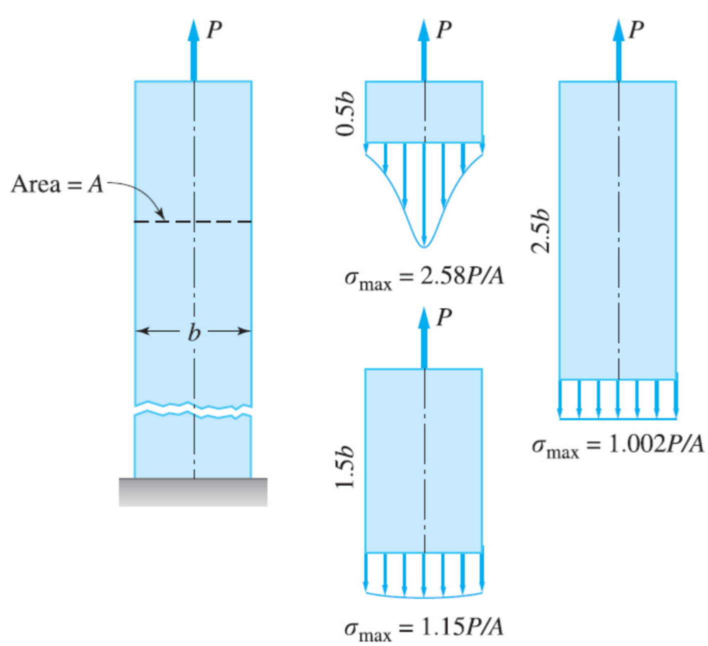
Stress distributions at different sections along axially loaded member



Centric Loading



Eccentric Loading

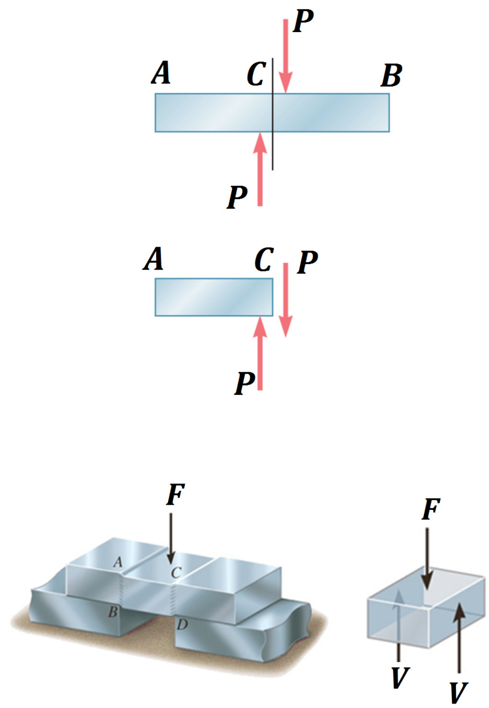


Shear Stress

The internal forces and the corresponding stresses discussed in previous section were normal to the section considered (normal stress). A very different type of stress is obtained when transverse forces **P** are applied to a member *AB*. Passing a section at *C* between the points of application of the two forces we obtain the diagram of portion *AC* shown. We conclude that internal forces must exist in the plane of the section, and that their resultant is equal to **P.** These elementary internal forces are called *shearing forces,* and the magnitude **P** of their resultant is the *shear* in the section. Dividing the shear **P** by the area *A* of the cross section, we obtain the *average shearing stress* in the section:

$$\tau_{avg} = \frac{P}{A}$$

It should be emphasized that the value obtained is an average value of the shearing stress over the entire section. Contrary to what we said earlier for normal stresses, the distribution of shearing stresses across the section cannot be assumed uniform. As you will see later, the actual value τ of the shearing stress varies from zero at the surface of the member to a maximum value τ_{max} that may be much larger than the average value τ_{avg}.

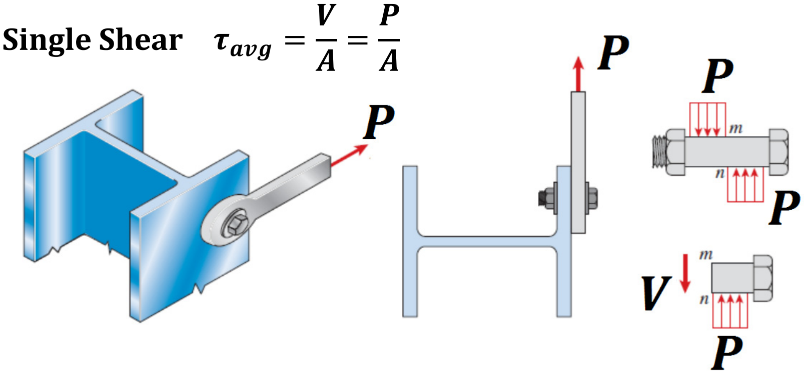


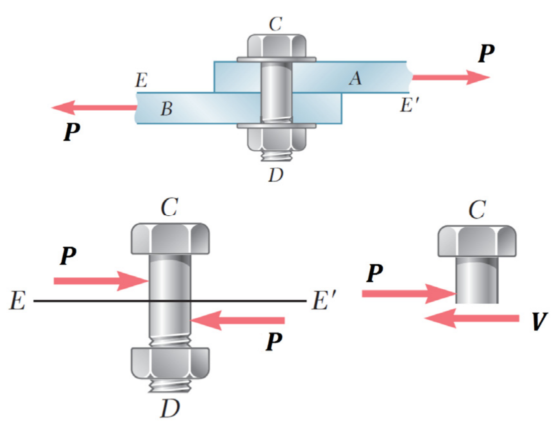
Member with transverse loads

Shear Stress in Connections and Joints

Shearing stresses are commonly found in bolts, pins, and rivets used to connect various structural members and machine components.

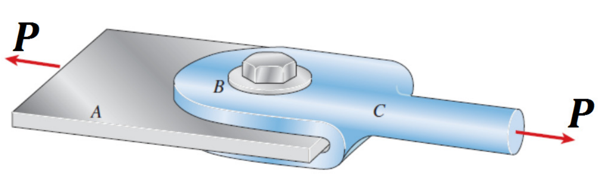
Single Shear $\tau_{avg} = \frac{V}{A} = \frac{P}{A}$

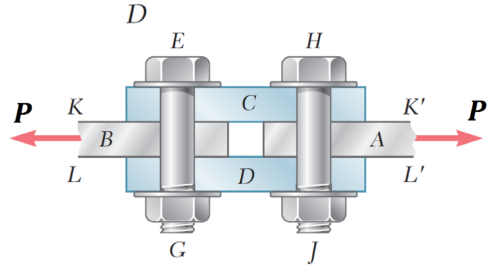


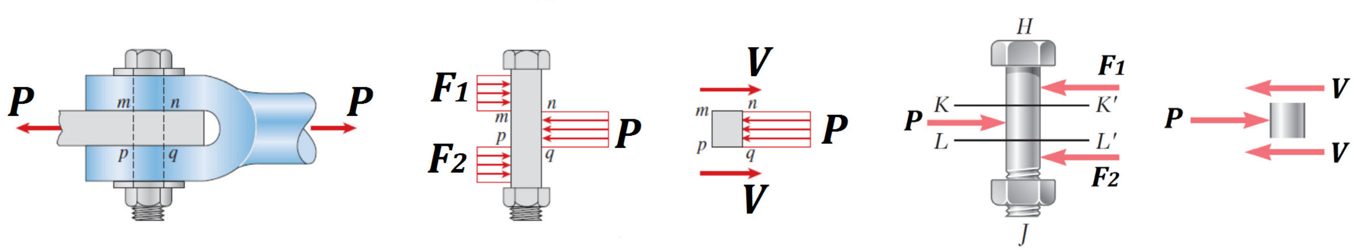


Double Shear

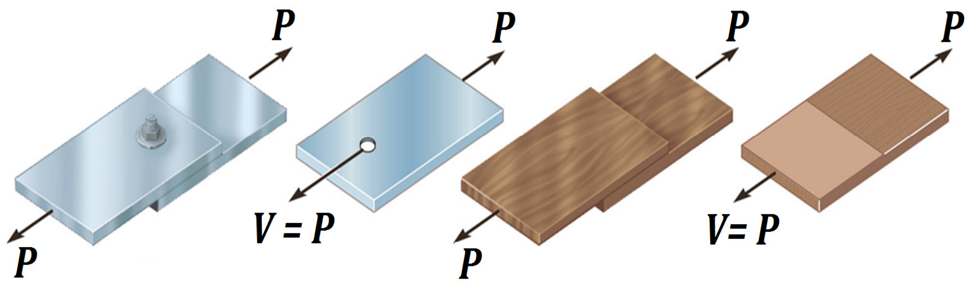
$$\tau_{avg} = \frac{V}{A} = \frac{P/2}{A}$$



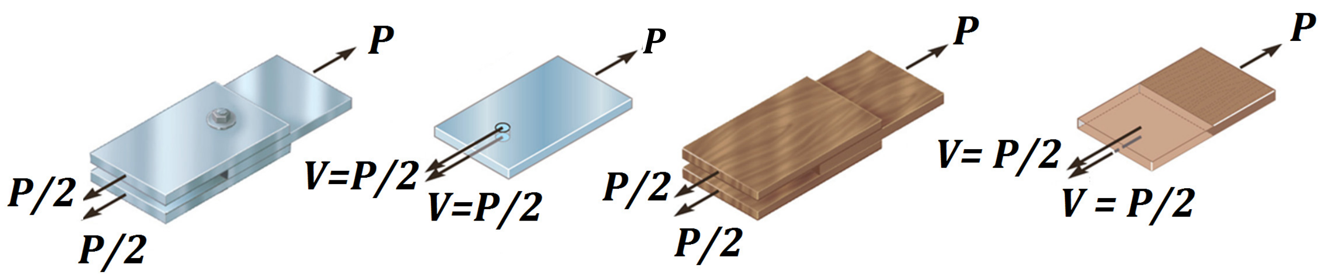




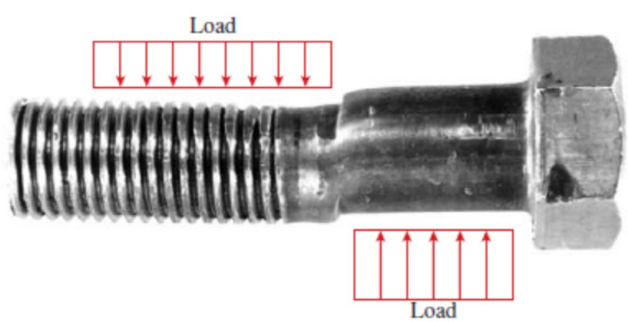
Other Examples of Single Shear



Other Examples of Double Shear



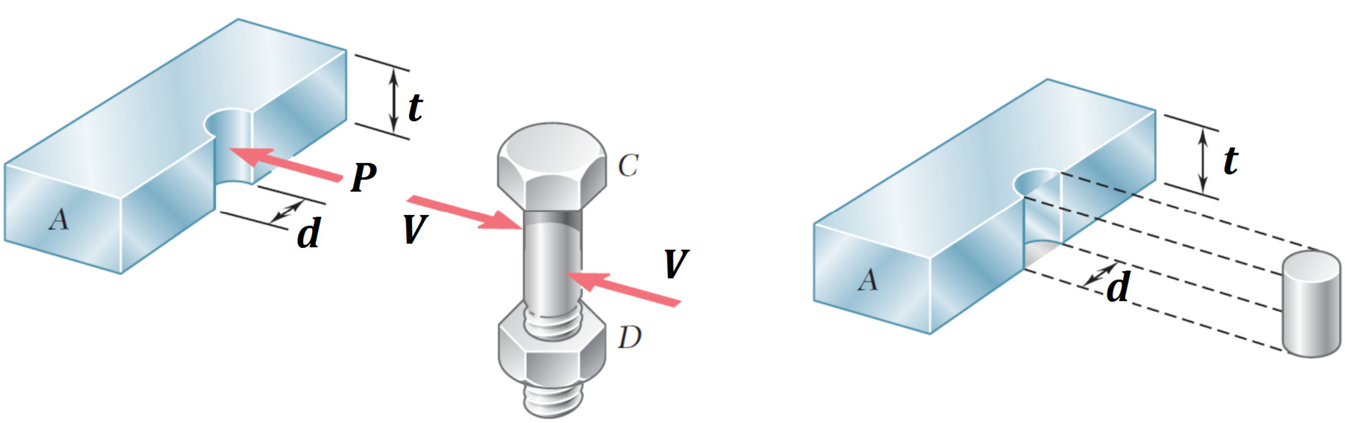
Failure of a Bolt



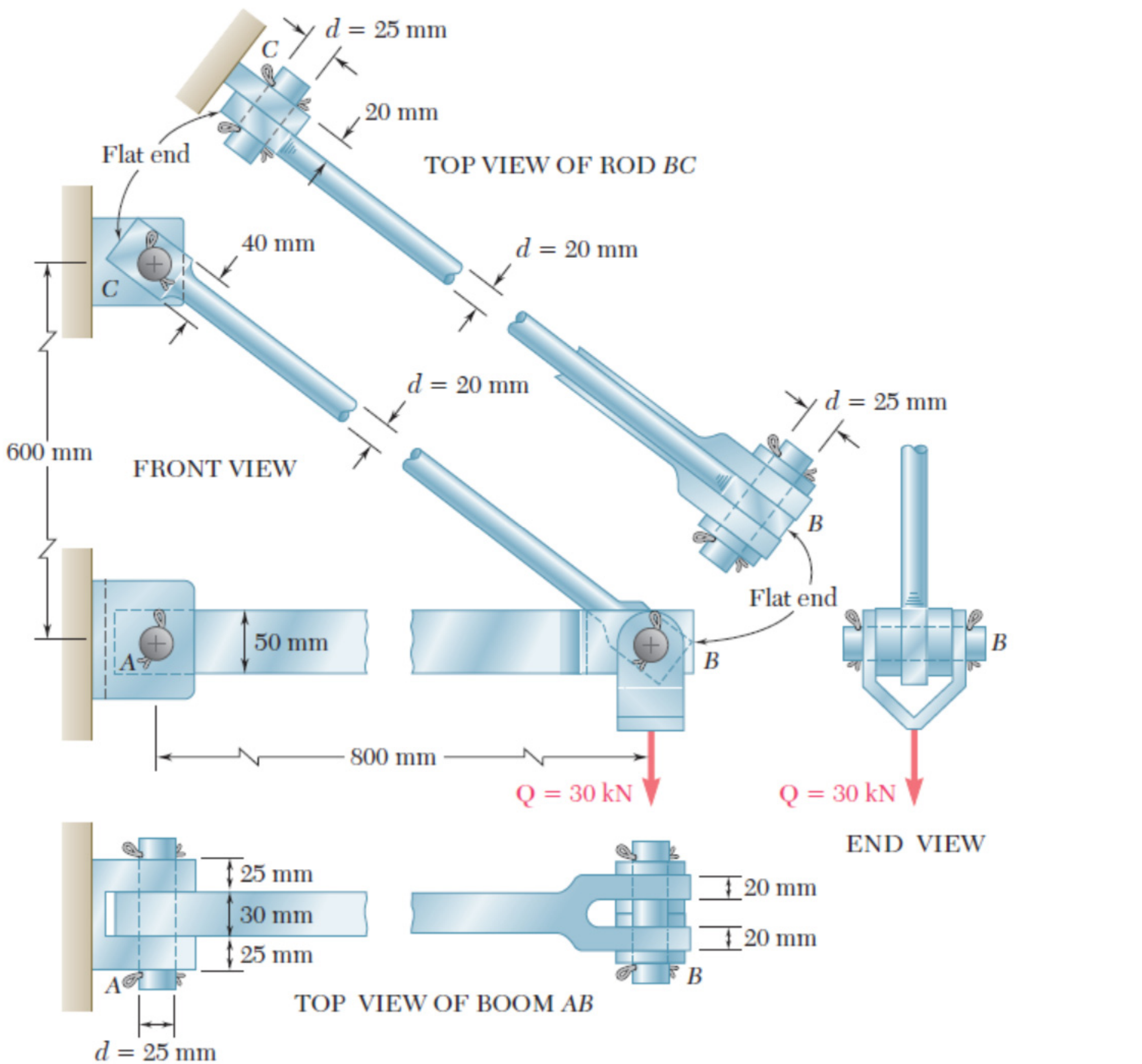
Bearing Stress in Connections

Bolts, pins, and rivets create stresses in the members they connect, along the *bearing surface,* or surface of contact.

$$\sigma_b = \frac{P}{A} = \frac{P}{td}$$



Example 4: Calculate normal, shear, and bearing stresses in the structure shown



From "Statics": $F_{AB} = 40\ kN\ (Compression)$

$$F_{BC} = 50\ kN\ (Tension)$$

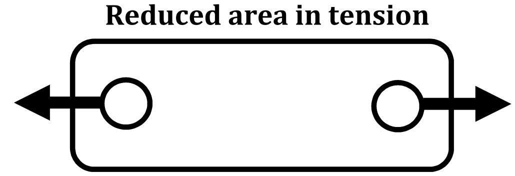
Normal stresses in rod BC:

$$\sigma_{BC}\ (at\ circular\ section) = \frac{50\ 000\ N}{\pi/4(20\ mm)^2} = 159.1\ MPa$$

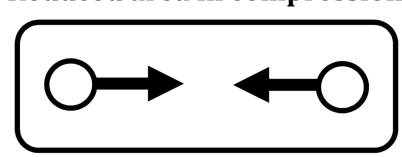
$$\sigma_{BC}\ (at\ flat\ section) = \frac{50\ 000\ N}{20\ mm \times 40\ mm} = 62.5\ MPa$$

$$\sigma_{BC}\ (at\ hole\ section) = \frac{50\ 000\ N}{20\ mm \times (40-25)\ mm} = 167\ MPa$$

Reduced area in tension



Reduced area in compression



Normal stresses in rod AB:

$$\sigma_{AB}\ (at\ mid\ section) = \frac{-40\,000\ N}{30\ mm \times 50\ mm} = -26.7\ MPa$$

$$\sigma_{AB}\ (at\ U\ shape\ part\ ) = \frac{-20\,000\ N}{20\ mm \times 50\ mm} = -20\ MPa$$

Shear stresses at connections

<u>Pin C:</u>

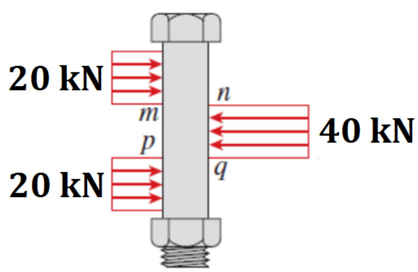
$$\tau_{avg} = \frac{P}{A} = \frac{50\,000\ N}{\pi/4(25\ mm)^2} = 102\ MPa$$

<u>Pin A:</u>

$$\tau_{avg} = \frac{P}{A} = \frac{20\,000\ N}{\pi/4(25\ mm)^2} = 40.7\ MPa$$

<u>Pin B:</u>

$$\tau_{avg)max} = \frac{P}{A} = \frac{25\,000\ N}{\pi/4(25\ mm)^2} = 50.9\ MPa$$



Bearing stresses

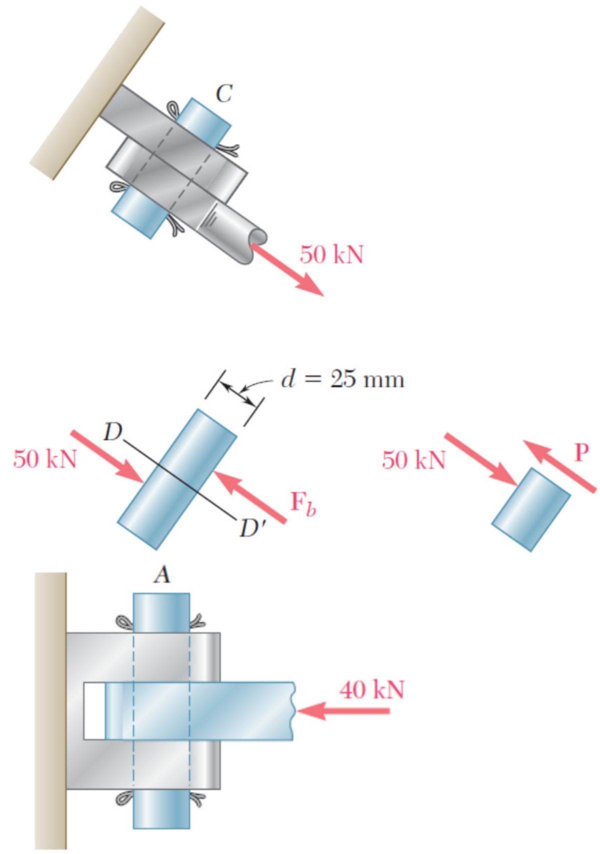
$$\sigma_b(at\ A) = \frac{P}{A} = \frac{P}{td} = \frac{40\,000\ N}{30\ mm\ \times 25\ mm} = 53.3\ MPa$$

$$\sigma_b(at\ barcket\ A) = \frac{P/2}{td} = \frac{20\,000\ N}{25\ mm\ \times 25\ mm} = 32\ MPa$$

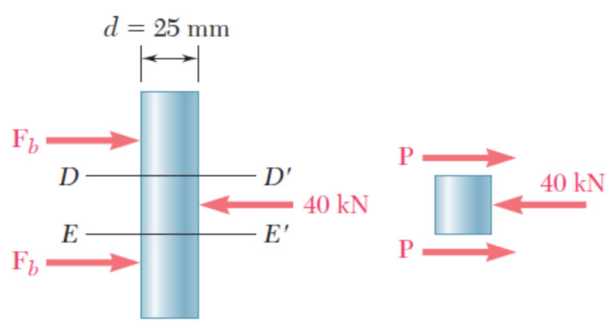
$$\sigma_b(at\ C) = \frac{P}{A} = \frac{P}{td} = \frac{50\,000\ N}{20\ mm\ \times 25\ mm} = 100\ MPa$$

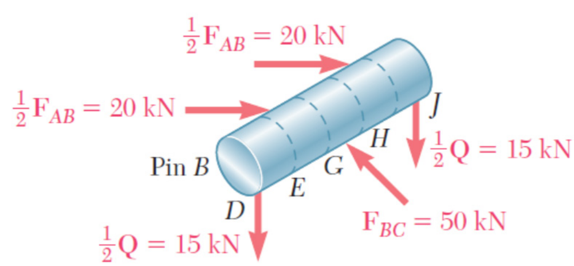
$$\sigma_b(at\ barcket\ C) = \frac{P}{td} = \frac{50\,000\ N}{20\ mm\ \times 25\ mm} = 100\ MPa$$

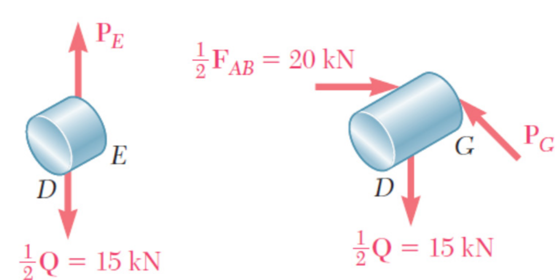
The same procedure should be followed for bearing stresses at pin B.



(a)

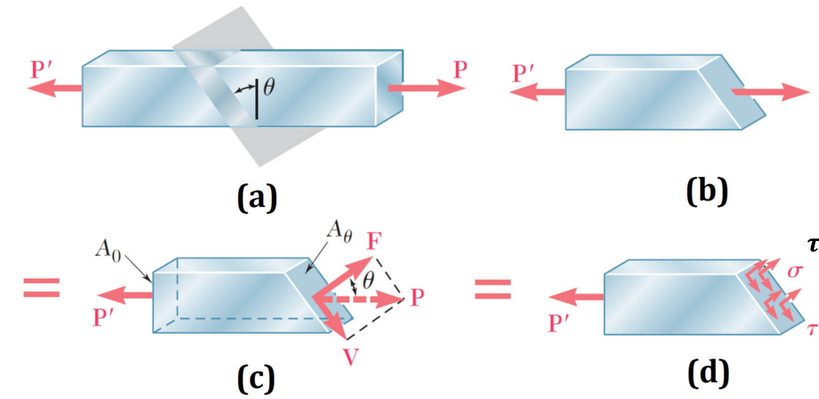






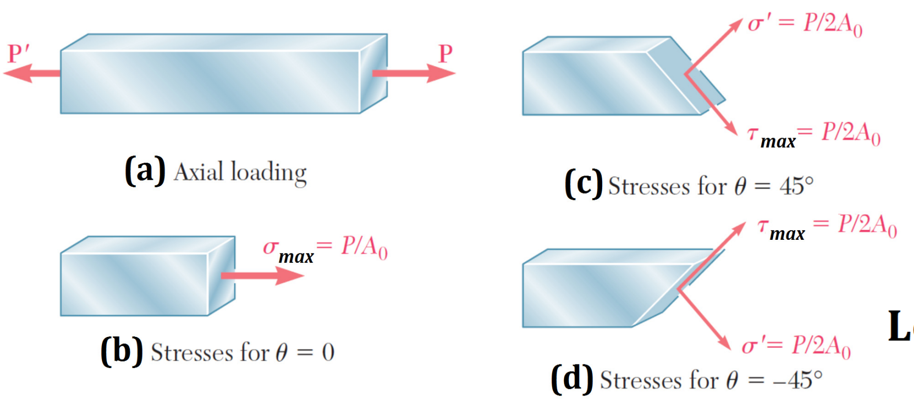
Stress on an Oblique Plane under Axial Loading

Axial forces cause both normal and shearing stresses on planes which are not ***perpendicular*** to the axis of the member. Similarly, transverse forces exerted on a bolt or a pin cause both normal and shearing stresses on planes which are not ***perpendicular*** to the axis of the bolt or pin.



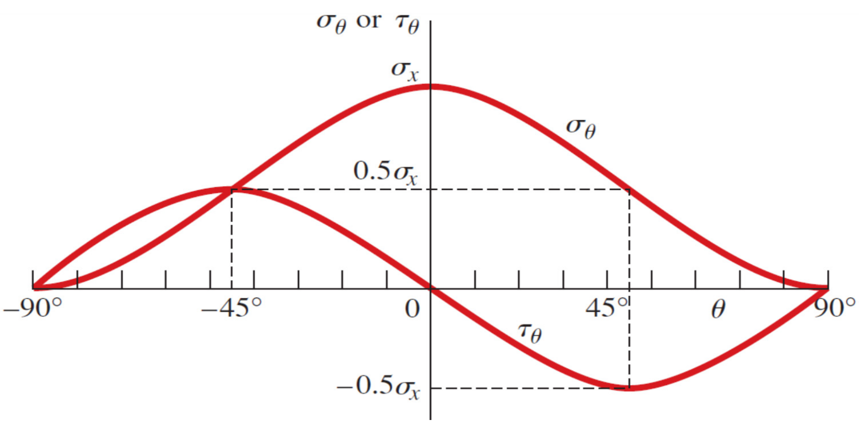
$$\sigma_\theta = \frac{F}{A_\theta} = \frac{P\cos\theta}{\dfrac{A_0}{\cos\theta}} = \frac{P}{A_0}\cos^2\theta = \sigma_x\cos^2\theta$$

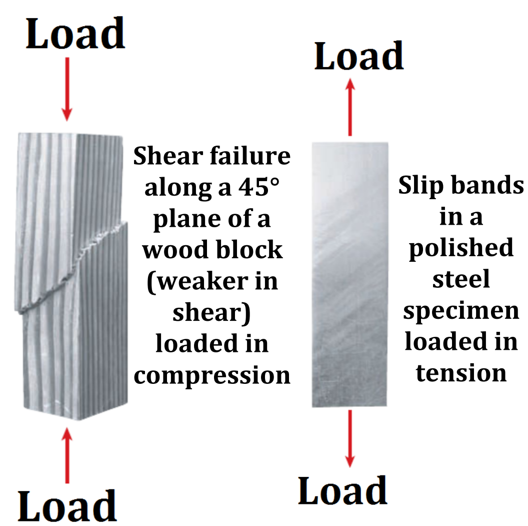
$$\tau_\theta = \frac{V}{A_\theta} = \frac{P\sin\theta}{A_0/\cos\theta} = \frac{P}{A_0}\sin\theta\cos\theta = \sigma_x\sin\theta\cos\theta$$

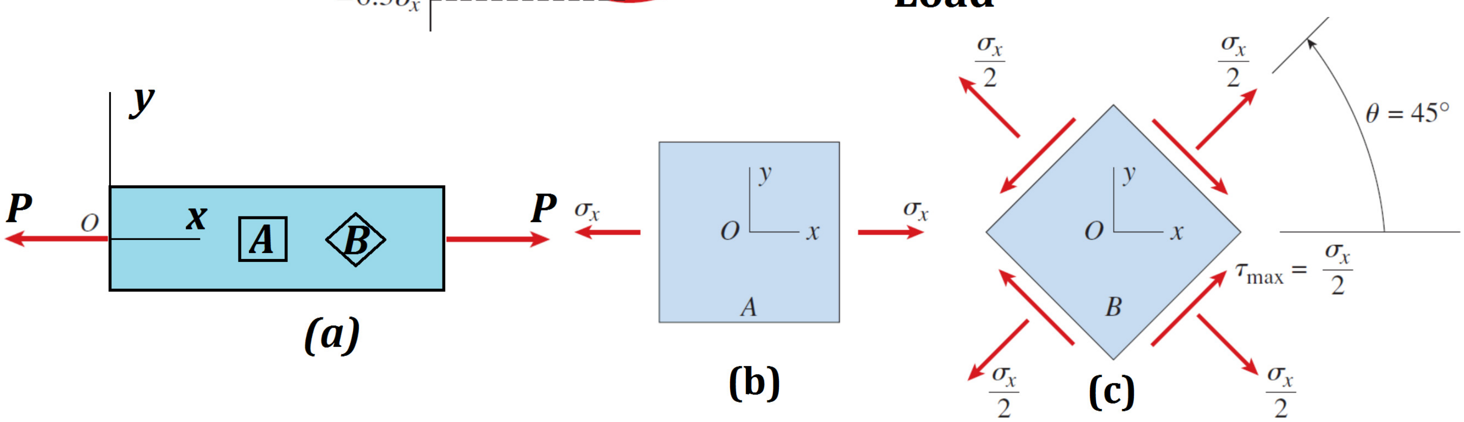


$$\sigma_{max} = \frac{P}{A_0}\ (at\ \theta = 0)\ \rightarrow \tau = 0$$

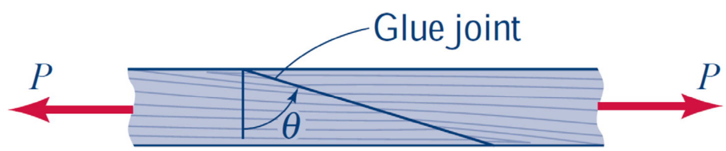
$$\tau_{max} = \frac{P}{2A_0}\ at\ \theta = 45° \ \rightarrow \sigma_{\theta=45°} = \frac{P}{2A_0}$$







Example 5: Several short pieces of timber are to be glued together end-to-end to form a single longer piece of timber as shown. The glue that is to be used in the splice joint is 50% stronger in shear than in tension. Is it possible to take advantage of this higher shear strength by selecting a splice angle θ such that the magnitude of the average shear stress on the joint is 50% higher than the average normal stress? If so, what is the appropriate angle?

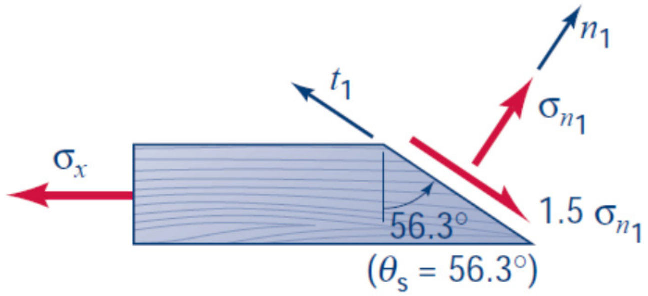


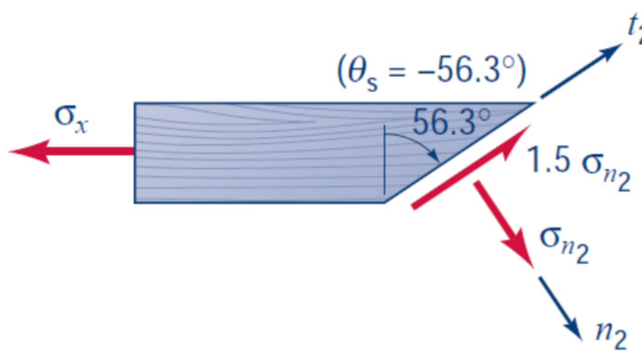
A diagonal splice joint.

We want to determine a splice angle θ_s such that $|\tau_{\theta s}| = 1.5\ \sigma_{\theta s}$

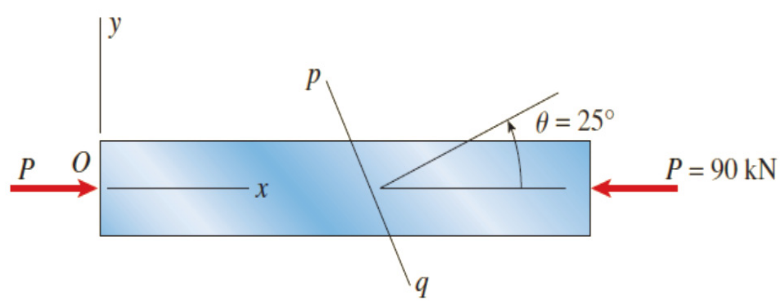
$$\rightarrow \sigma_x \sin\theta_s \cos\theta_s = \pm\ 1.5\sigma_x \cos^2\theta_s \rightarrow \tan\theta_s = \pm\ 1.5$$

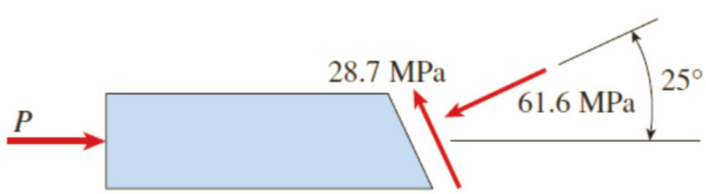
$$\rightarrow \theta_s = \pm 56.3°$$

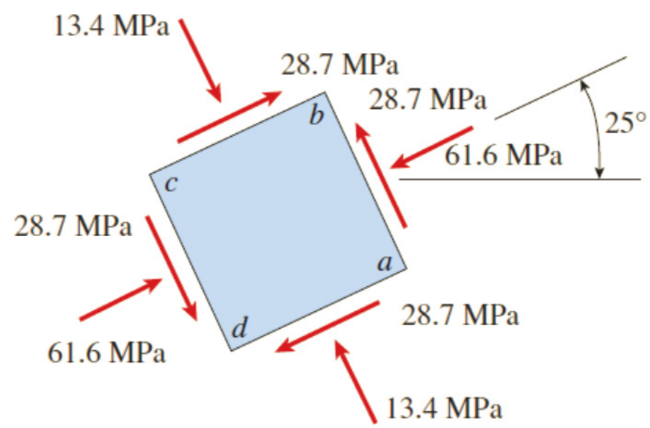




Example 6: A prismatic bar having cross-sectional area 1200 mm² is compressed by an axial load P = 90 kN. (a) Determine the stresses acting on an inclined section pq cut through the bar at an angle θ = 25°. (b) Determine the complete state of stress for θ = 25° and show the stresses on a properly oriented stress element.

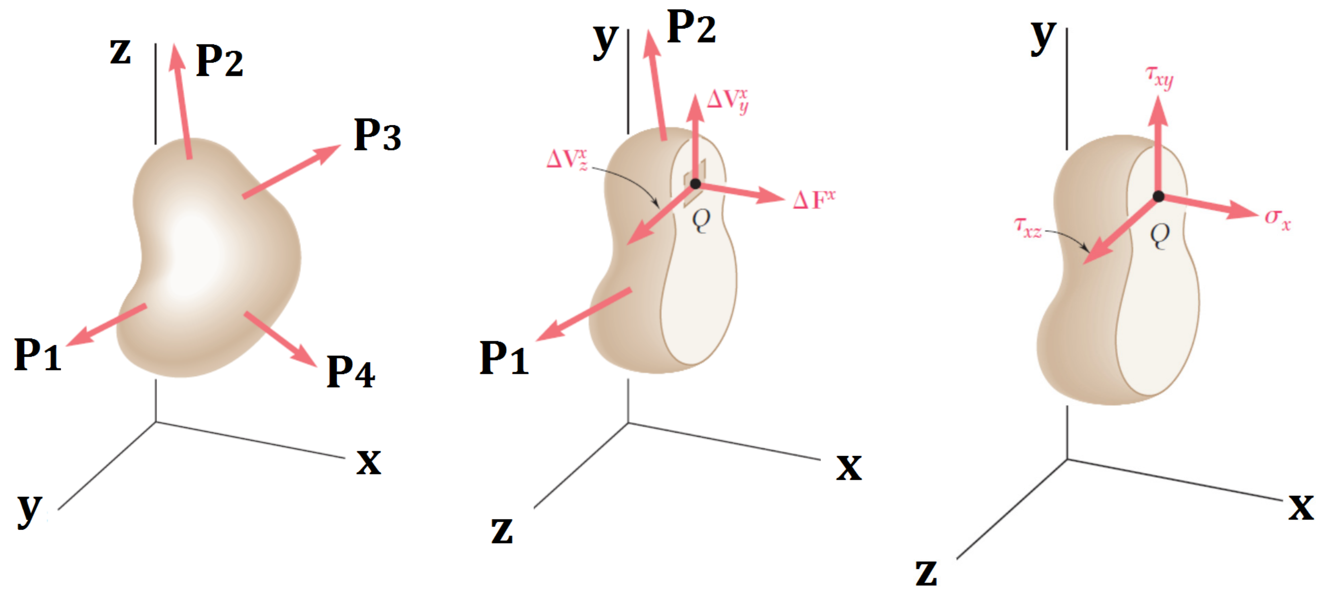




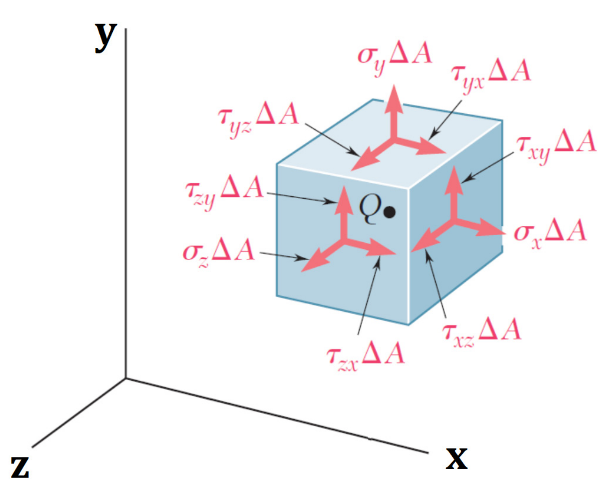


Stress under General Loading Conditions; Components of Stress

Thus far we have been limited to members under axial loading and connections under transverse loading. Most structural members and machine components are, however, under more complex loading conditions. We are not trying to determine other stress components here but simply aim to introduce them and the convections involved.

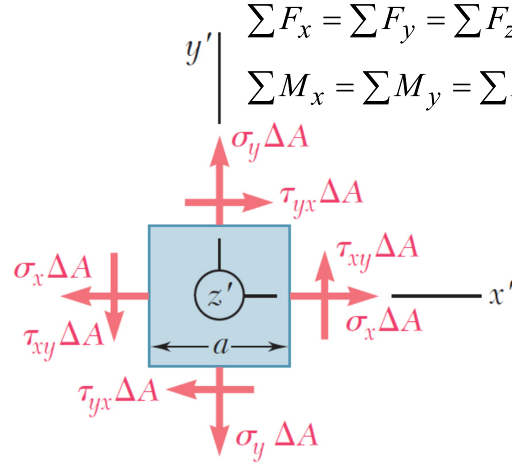


$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F^x}{\Delta A} \qquad \tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta V_y^x}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta V_z^x}{\Delta A}$$



$$\sum F_x = \sum F_y = \sum F_z = 0$$

$$\sum M_x = \sum M_y = \sum M_z = 0$$



$$\sum \boldsymbol{M_z = 0 \rightarrow (\tau_{xy} \Delta A) a - (\tau_{yx} \Delta A) a = 0}$$

$$\boldsymbol{\tau_{xy} = \tau_{yx} \rightarrow \textit{similarly } \tau_{xz} = \tau_{zx} \textit{ and } \tau_{yz} = \tau_{zy}}$$

$$\text{Tensor of stress at a point } = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

In general loading conditions six independent stress components at each point may exist.

Design Considerations

(1) Ultimate Strength of a Material

An important element to be considered by a designer is how the material that has been selected will behave under a load. For a given material, this is determined by performing specific tests on prepared samples of the material. For example, a test specimen of steel may be prepared and placed in a laboratory testing machine to be subjected to a known centric axial tensile force, as it will be described later. As the magnitude of the force is increased, various changes in the specimen are measured, for example, changes in its length and its diameter. Eventually the largest force which may be applied to the specimen is reached, and the specimen breaks. This largest force is called the ultimate load for the test specimen and is denoted by P_U. Since the applied load is centric, we may divide the ultimate load by the original cross-sectional area of the rod to obtain the ultimate normal stress of the material used. This stress is also known as the ***ultimate strength*** in tension of the material. Several test procedures are available to determine the ultimate shearing stress, or ultimate strength in shear, of a material (will be explained later on).

(2) Factor of Safety

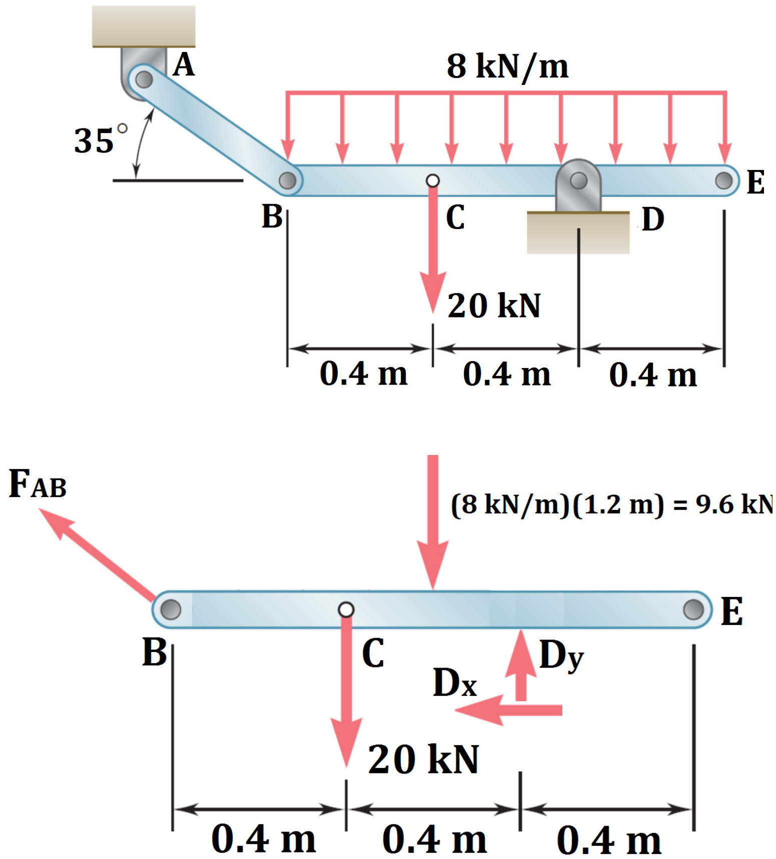
The maximum load that a structural member or a machine component will be allowed to carry under normal conditions of utilization is considerably smaller than the ultimate load. This smaller load is referred to as the allowable load. Thus, only a fraction of the ultimate-load capacity of the member is utilized when the allowable load is applied. The remaining portion of the load-carrying capacity of the member is kept in reserve to assure its safe performance. The ratio of the ultimate load to the allowable load is used to define the factor of safety:

$$\boldsymbol{Factor\ of\ Safety = F.S. = \frac{Ultimate\ Load}{Allowable\ Load}\ \ or\ \ \frac{Ultimate\ Stress}{Allowable\ Stress}}$$

Of course, the factor of safety must be greater than 1.0 if failure is to be avoided. Depending upon the circumstances, factors of safety from slightly above 1.0 to as much as 10 are used. The use of a small value of factor of safety (e.g., FS = 1.1) is justified only when it is possible, by analysis and testing, to sufficiently minimize uncertainties, and when there is no likelihood that failure will result in unacceptable circumstances such as serious personal injury or death. On the other hand, it is undesirable to use a factor of safety that is unnecessarily large (e.g., FS = 3), since that would lead to excess structural weight, which, in turn, entails excess initial costs and operating costs. Since the choice of a value of factor of safety has such important economic and legal implications, design specifications, including the relevant factor(s) of safety to be used, conform to design codes or other standards developed by groups of experienced engineers in engineering societies or in various government agencies.

(14)

Example 7: Link *AB* is to be made of a steel for which the ultimate normal stress is 450 MPa. Determine the cross-sectional area of *AB* for which the factor of safety will be 3.50. Assume that the link will be adequately reinforced around the pins at *A* and *B*. Design the single shear pin A (find its diameter) assuming a factor of safety of 2.5 if its ultimate shear stress is 100 MPa.



$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} \rightarrow \frac{450\ MPa}{3.5} = 128.57\ MPa$$

$$\sum M_D = 0 \rightarrow$$

$$F_{AB}\ \sin 35° \times 0.8\ m - 20\ kN \times 0.4\ m - 9.6\ kN \times 0.2\ m = 0.$$

$$F_{AB} = 21.618\ kN, \qquad \frac{F_{AB}}{A_{AB}} = 128.57\ MPa$$

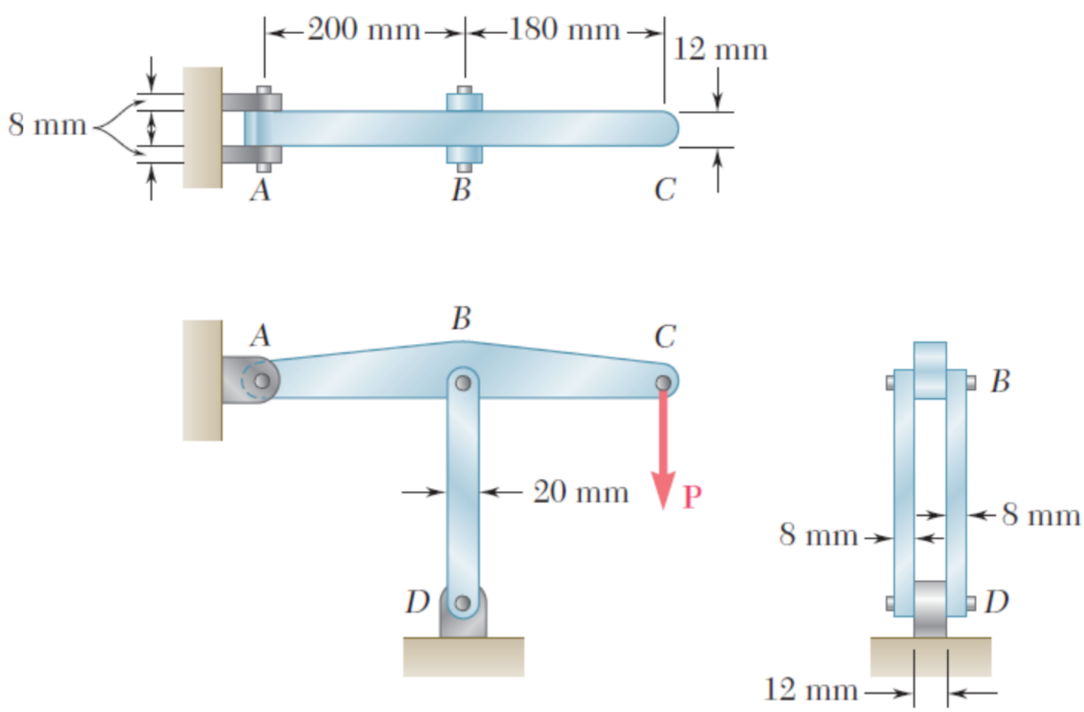
$$\rightarrow \frac{21618\ N}{A_{AB}} = 128.57\ MPa \rightarrow A_{AB} = 168.1\ mm^2$$

$$\tau_A = \frac{F_{AB}}{\frac{\pi}{4}d^2} = \frac{21618\ N}{\frac{\pi}{4}d^2} = \frac{100\ MPa}{2.5}$$

$$d = 26.2\ mm$$

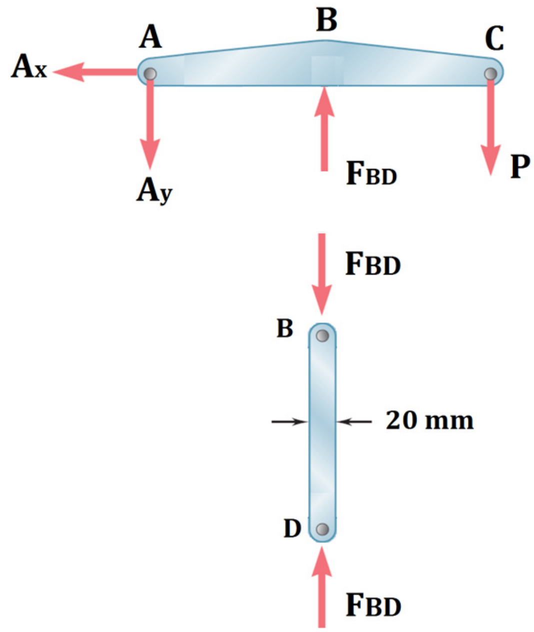
TBR 1 (1390): In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections (pins) and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load **P** if an overall factor of safety of 3.0 is desired. Find bearing stress at D based on the calculated **P**.

Top View



Front View **Side View**

Based on compression in links BD:



$$\sigma_{BD} = \frac{F_{BD}/2}{A_{BD}} \rightarrow \frac{250\ MPa}{3} = \frac{F_{BD}/2}{20\ mm \times 8\ mm}$$

$$F_{BD} = 26666.67\ N$$

$$\sum M_A = 0 \rightarrow F_{BD} \times 200\ mm - P \times 380\ mm = 0$$

$$\rightarrow P = \frac{10}{19} F_{BD} = \frac{10}{19} \times 26666.67\ N = 14035.1\ N$$

Based on double shear in pin A:

$$\sum F_x = 0 \rightarrow A_x = 0$$

$$\tau = \frac{A_y/2}{\frac{\pi}{4} d^2} \rightarrow \frac{100\ MPa}{3} = \frac{A_y/2}{\frac{\pi}{4}(8\ mm)^2} \rightarrow A_y = 3351\ N$$

$$\sum M_B = 0 \rightarrow P(180) - A_y(200) = 0 \rightarrow A_y = \frac{9}{10} P \qquad \rightarrow \quad P = \frac{10}{9} \times 3351\ N = 3723.4\ N$$

Based on double shear in pins B and D:

$$\tau = \frac{F_{BD}/2}{\frac{\pi}{4} d^2} \rightarrow \frac{100\ MPa}{3} = \frac{F_{BD}/2}{\frac{\pi}{4}(12\ mm)^2} \rightarrow F_{BD} = 8377.6\ N \rightarrow P = \frac{10}{19}(8377.6\ N) = 3968.3\ N$$

$$P = \min(14035.1\ N, 3723.4\ N, 3968.3\ N) = 3723.4\ N$$

Bearing stresses at D:

$$\sigma_B(at\ link\ BD) = \frac{\frac{F_{BD}}{2}}{8\ mm \times 12\ mm} = \frac{\frac{\frac{19}{10}P}{2}}{8\ mm \times 12\ mm} = \frac{\frac{\frac{19}{10}(3723.4\ N)}{2}}{8\ mm \times 12\ mm} = 36.8\ MPa$$

$$\sigma_B(at\ the\ bracket\ D) = \frac{F_{BD}}{12\ mm \times 12\ mm} = \frac{\frac{19}{10}P}{12\ mm \times 12\ mm} = \frac{\frac{19}{10}(3723.4\ N)}{12\ mm \times 12\ mm} = 49.1\ MPa$$

TBR 2: The steel plane truss shown in the figure is loaded by three forces *P*, each of which is 490 kN. The truss members each have a cross-sectional area of 3900 mm² and are connected by pins each with a diameter of d_p =18 mm. Members *AC* and *BC* each consist of one bar with thickness of t_{AC} = t_{BC} =19 mm. Member *AB* is composed of two bars [see figure part (b)] each having thickness $t_{AB}/2$ =10 mm and length *L* = 3 m. The roller support at *B*, is made up of two support plates, each having thickness $t_{sp}/2$ =12 mm.

(a) Find support reactions at joints *A* and *B* and forces in members *AB, BC*, and *AB*.

(b) Calculate the largest average shear stress in the pin at joint *B*, disregarding friction between the members; see figures parts (b) and (c) for sectional views of the joint.

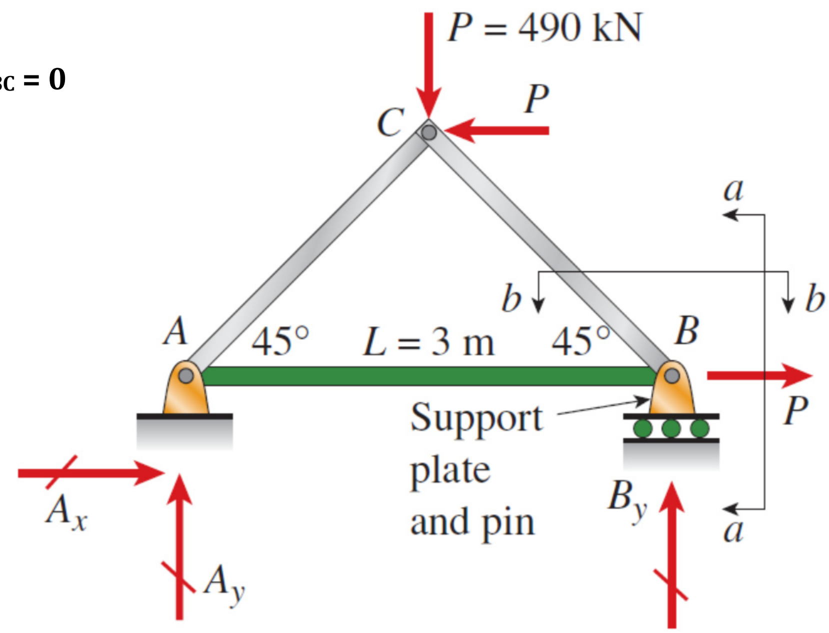
(c) Calculate the largest average bearing stress acting against the pin at joint *B*.

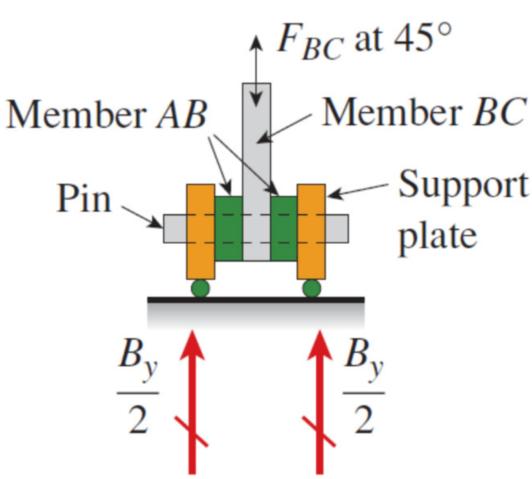
Answer:

(a) F_{AC} = -693 kN, F_{AB} = 490 kN, F_{BC} = 0

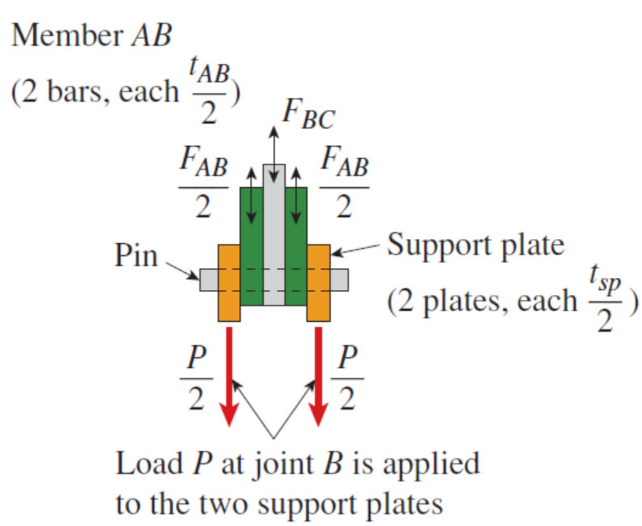
(b) 963 MPa

(c) 1361 MPa





(b) Section *a–a* at joint *B* (Elevation view)



(c) Section *b–b* at joint *B* (Plan view)

Solution (TBR 2):

NUMERICAL DATA

$L = 3000 \text{ mm} \qquad P = 490 \text{ kN}$

$d_p = 18 \text{ mm} \qquad A = 3900 \text{ mm}^2$

$t_{AC} = 19 \text{ mm} \qquad t_{BC} = t_{AC}$

$t_{AB} = 20 \text{ mm} \qquad t_{sp} = 24 \text{ mm}$

(a) SUPPORT REACTIONS AND MEMBER FORCES

$\sum F_x = 0 \qquad A_x = 0 \quad \leftarrow$

$$\sum M_A = 0 \qquad B_y = \frac{1}{L}\left(P\frac{L}{2} - P\frac{L}{2}\right)$$

$B_y = 0 \quad \leftarrow$

$\sum F_y = 0 \qquad A_y = P$

$A_y = 490 \text{ kN} \quad \leftarrow$

METHOD OF JOINTS

$F_{AB} = P \qquad F_{BC} = 0 \quad \leftarrow$

$F_{AC} = -\sqrt{2}P$

$F_{AB} = 490 \text{ kN} \quad \leftarrow$

$F_{AC} = -693 \text{ kN} \quad \leftarrow$

(b) MAX. SHEAR STRESS IN PIN AT B

$$A_s = \frac{\pi d_p^2}{4} \qquad A_s = 254.469 \text{ mm}^2$$

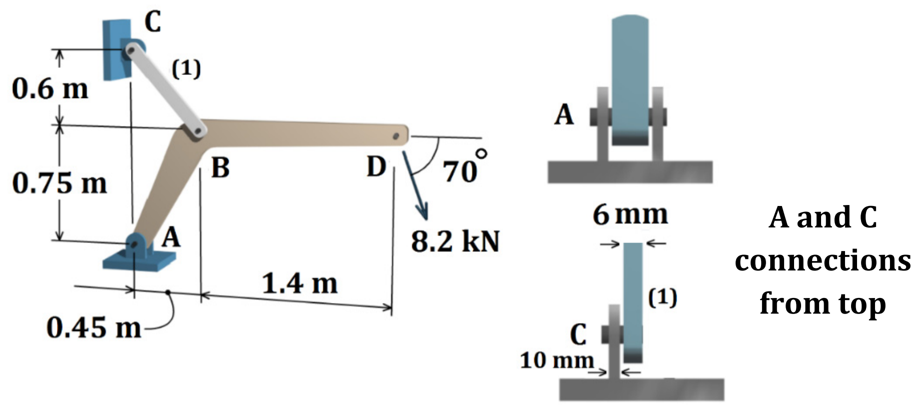
$$\tau_{p\max} = \frac{\frac{F_{AB}}{2}}{A_s} \qquad \tau_{p\max} = 963 \text{ MPa} \quad \leftarrow$$

(c) MAX. BEARING STRESS IN PIN AT B ($t_{ab} < t_{sp}$ SO BEARING STRESS ON AB WILL BE GREATER)

$$A_b = d_p \frac{t_{AB}}{2}$$

$$\sigma_{b\max} = \frac{\frac{F_{AB}}{2}}{A_b} \qquad \sigma_{b\max} = 1361 \text{ MPa} \quad \leftarrow$$

TBR 3 (1391): Rigid bar *ABD* is supported by a pin connection at *A* and a tension link *BC*. The 8-mmdiameter pin at *A* is supported in a double shear connection, and the 12-mm-diameter pins at *B* and *C* are both used in single shear connections. Link *BC* is 30-mm wide and 6-mm thick. The ultimate shear strength of the pins is 330 MPa and the yield strength of link *BC* is 250 MPa. (a) Determine the factor of safety in pins *A* and *B* with respect to the ultimate shear strength. (b) Determine the factor of safety in link *BC* with respect to the yield strength. (c) Determine bearing stresses at C.



(1) $\Sigma M_A = 0 \circlearrowleft$

$8.2^{kN}\cos 70^\circ (0.75^m) + 8.2^{kN}\sin 70^\circ (1.85^m) - F_1 \cos 53.13^\circ (0.75^m) - F_1 \sin 53.1^\circ (0.45^m) = 0.$

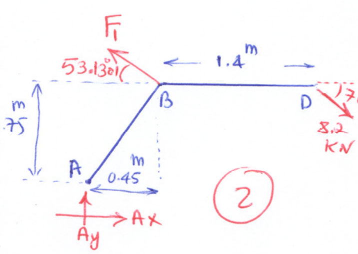
$\rightarrow F_1 = 20.19^{kN}$ (2)

(2) $\Sigma F_x \rightarrow = 0 \Rightarrow A_x + 8.2\cos 70^\circ - F_1 \cos 53.13^\circ = 0$

$\rightarrow A_x = 9.31\ kN$ (2)

(3) $\Sigma F_y \uparrow = 0. \rightarrow A_y + F_1 \sin 53.13^\circ - 8.2 \sin 70^\circ = 0.$

$\rightarrow A_y = -8.45\ kN$ (2) $\rightarrow A = \sqrt{A_x^2 + A_y^2} = 12.57^{kN}$ (1)

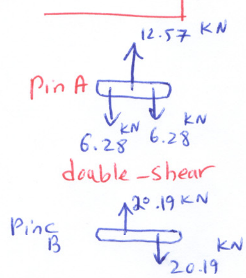


FBD of ABD

Pin shear stresses

Pin A : $\rightarrow \tau = \dfrac{6.28\times10^3\ N}{\frac{\pi}{4}(8\ mm)^2} \cong 125^{MPa}$ (2)

Pins B and C : $\rightarrow \tau = \dfrac{(20.19\times10^3 N)}{\pi/4\ (12\ mm)^2} \cong 178.5^{MPa}$ (2)



$F.S._{Pin\ A} = \dfrac{330^{MPa}}{125\ MPa} \cong 2.64$ (1)

$F.S._{Pins\ B,C} = \dfrac{330\ MPa}{178.5} = 1.84$ (1)

Normal stress in link BC

$\sigma_{BC} = \dfrac{20.19\times10^3\ N}{6\times30\ mm^2} = 112.2^{MPa}$ (2)

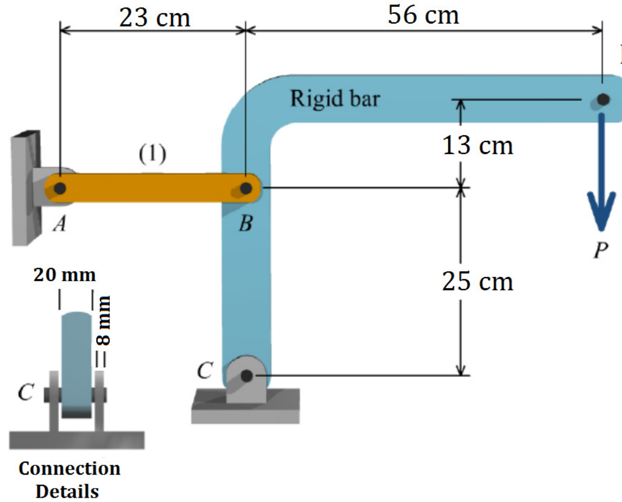
$F.S._{BC} = \dfrac{250\ MPa}{112.2\ MPa} = 2.23$ (1)

Bearing stresses

$\sigma_B)_{CB} = \dfrac{20.19\times10^3 N}{6\times12\ mm^2} = 280.4^{MPa}$ (1)

$\sigma_B)_{Bracket} = \dfrac{20.19\times10^3\ N}{10\times12\ mm^2} = 168.25^{MPa}$ (1)

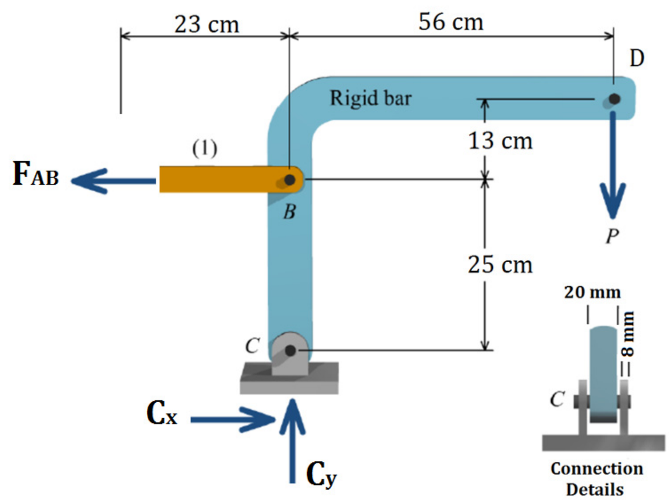
TBR 4 (1392): The simple pin-connected structure carries a concentrated load P as shown. The rigid bar is supported by strut AB and by a pin support at C. The steel strut AB has a cross sectional area of 161 mm^2 and a yield strength of 413 MPa. The diameter of the steel pin at C is 9.5 mm, and the ultimate shear strength is 372 MPa. If a factor of safety of 2.0 is required in both the strut and the pin at C, determine the maximum load P that can be supported by the structure. Based on the calculated P, determine bearing stresses at C.



$$\sigma_{AB} = \frac{413\ MPa}{2} = \frac{F_{AB}}{A_{AB}} = \frac{F_{AB}}{161\ mm^2} \rightarrow F_{AB} = 33246.5\ N$$

$$\sum M_C = 0 \rightarrow P(56\ cm) - F_{AB}(25\ cm) = 0$$

$$\rightarrow P = \frac{25}{56}(33246.5) = 14842.2\ N \qquad (1)$$



$$\tau_C = \frac{372\ MPa}{2} = \frac{C/2}{\frac{\pi}{4}(9.5\ mm)^2} \rightarrow C = 26368.17\ N$$

$$\sum F_y = 0 \rightarrow C_y = P$$

$$\sum M_B = 0 \rightarrow C_x(25\ cm) - P(56) = 0 \rightarrow C_x = 2.24P$$

$$\sqrt{C_x^2 + C_y^2} = C = 26368.17\ N \rightarrow \sqrt{(2.24P)^2 + (P)^2} = 26368.17\ N$$

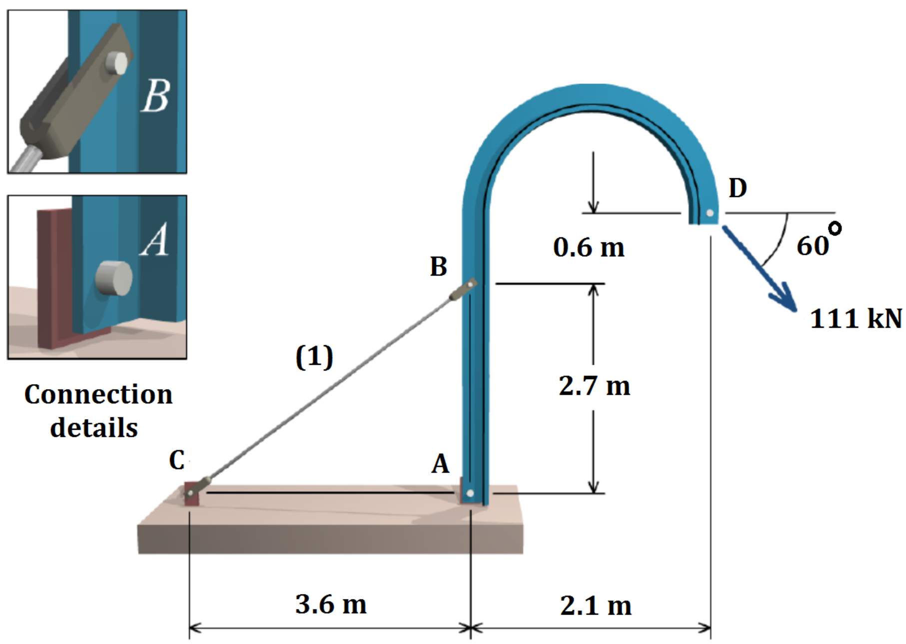
$$\rightarrow P = 10749\ N \qquad (2)$$

$$P_{max} = 10749\ N = 10.75\ kN$$

$$\sigma_b at\ BC = \frac{26368.17\ N}{20 \times 9.5\ mm^2} = 138.7\ MPa$$

$$\sigma_b at\ Bracket = \frac{\frac{26368.17}{2} N}{8 \times 9.5\ mm^2} = 346.9\ MPa$$

TBR 4 (1393): The davit *ABD* with a cross sectional area of 967 mm^2 is supported at *A* by a pin connection and at *B* by a tie rod (1). The pin at *A* has a diameter of 32 mm and the pins at *B* and *C* are each 19 mm diameter pins. The ultimate shear strength in each pin is 550 MPa, and the yield strength of the tie rod is 248 MPa. A concentrated load of 111 kN is applied as shown to the davit structure at *D*. Determine: (a) the factor of safety with respect to the yield strength for tie rod (1), (b) factor of safety with respect to the ultimate strength for the pins at *A* and *B*, and (c) bearing stress at bracket A (assuming a thickness of 30 mm).



$$\sum M_A = 0 \rightarrow F_1 \cos 36.8° \times 2.7 = 111 \cos 60° \times (2.7 + 0.6) + 111 \sin 60° \times 2.1 \rightarrow$$

$$F_1 = 178.25\ kN \quad 3$$

$$\sum F_x = 0 \rightarrow A_x + 111 \cos 60° - 178.25 \times \cos 36.8° = 0 \rightarrow A_x = 87.1\ kN \quad 3$$

$$\sum F_y = 0 \rightarrow A_y - 111 \sin 60° - 178.25 \times \sin 36.8° = 0 \rightarrow A_y = 203.1\ kN \quad 3$$

$$A = \sqrt{A_x^2 + A_y^2} = 220.96\ kN \quad 3$$

$$\sigma_1 = \frac{178\,250\ N}{967\ mm^2} = 184.3\ MPa \rightarrow FS_1 = \frac{248}{184.3} = 1.34 \quad 3$$

$$\tau_B = \frac{\frac{178\,250\ N}{2}}{\pi\left(\frac{19\ mm}{2}\right)^2} = 314.3\ MPa \rightarrow FS_B = \frac{550}{314.3} = 1.75 \quad 3$$

$$\tau_A = \frac{220\,960\ N}{\pi\left(\frac{32\ mm}{2}\right)^2} = 274.8\ MPa \rightarrow FS_B = \frac{550}{274.8} = 2.0 \quad 3$$

$$\sigma_B at\ Bracket\ A = \frac{220\,960\ N}{30\ mm\ \times 32\ mm} = 230.2\ MPa \quad 4$$